CETUP*-12/013
FERMILAB-CONF-12-527-T

An Explicit SU(12) Family and Flavor Unification Model

Carl H. Albright*,†, Robert P. Feger** and Thomas W. Kephart**

*Department of Physics, Northern Illinois University, DeKalb, IL 60115
†Theoretical Physics, Fermilab, Batavia, IL 60510
**Department of Physics and Astronomy, Vanderbilt University, Nashville, TN 37235

Abstract. An explicit SUSY SU(12) unification model with three light chiral families is presented which avoids any external flavor symmetries. The hierarchy of quark and lepton masses and mixings is explained by higher dimensional Yukawa interactions involving Higgs bosons containing SU(5) singlet fields with VEVs appearing at or below the SUSY GUT scale of 2×10^{16} GeV, approximately 50 times smaller than the SU(12) unification scale. The model has been found to be in good agreement with the observed quark and lepton masses and mixings, with nearly all prefactors of $\mathcal{O}(1)$ in the four Dirac and one Majorana fermion mass matrices.

Keywords: family and flavor unification, SU(12)
PACS: 12.10.Dm, 12.15.Ff, 14.60.Pq

MOTIVATION

Grand Unified Theories (GUTs) of quarks and leptons have played an important role for almost 40 years in theoretical attempts to make sense of the apparent group structure and mass spectra of the quark and lepton matter fields and coupling strengths of the gauge and Higgs fields observed in Nature. Although many varieties of models have been constructed, the most popular unified ones are based on the unitary, orthogonal, or exceptional groups SU(5), SO(10), or E_6, respectively. But in each of these cases, the chiral irreducible representations (irreps) can uniquely describe only one family of quarks and leptons: towit, **10**, $\overline{\mathbf{5}}$, and **1** for SU(5), **16** for SO(10), and **27** for E_6. In order to accommodate the three families observed todate, it has been conventional to introduce in addition to one of the above $G_{\rm family}$ groups, a G_{flavor} symmetry group which also distinguishes the families. While continuous flavor symmetries such as U(1), SU(2), SU(3) and their products have been considered in the past, more recently discrete symmetry groups such as A_4, T' and S_4, etc. have been fashionable in the past 10 years [1, 2]. In either case, the GUT model then involves the direct product group $G_{family}\times G_{flavor}$.

True family and flavor unification requires the introduction of a higher rank simple group. Some earlier studies along this line have been based on SO(18) [3, 4], SU(11) [5, 6], and SU(9) [7, 8]. More recently, models based on SU(7) [9], SU(8) [10], and SU(9) again [11, 12], the latter reference by two of us (RF and TWK), but none have been totally satisfactory due to a huge number of unwanted states and/or unsatisfactory mass matrices. Here we describe an SU(12) unification model [13] with interesting features that was constructed with the help of a Mathematica computer package called LieART written by two of us (RPF and TWK) [14]. This program allows one to compute tensor products, branching rules, etc., and perform detailed searches for satisfactory models in a timely fashion. While other smaller and larger rank unitary groups were examined, a model based on SU(12) appeared to be the most satisfactory minimal one for our purpose. We sketch here the model construction and point out that further details can be found in [13].

SU(12) UNIFICATION MODEL AND PARTICLE ASSIGNMENTS

While the three popular GUT groups cited earlier each have just one useful chiral irreducible representation (irrep), SU(12) has 11 totally antisymmetric irreps: **12**, **66**, **220**, **495**, **792**, **924**, $\overline{\mathbf{792}}$, $\overline{\mathbf{495}}$, $\overline{\mathbf{220}}$, $\overline{\mathbf{66}}$, and $\overline{\mathbf{12}}$, of which 10 are complex (while **924** is real), which allow three SU(5) families to be assigned to different chiral irreps. For this purpose, one chooses an anomaly-free set of SU(12) irreps which contains three chiral SU(5) families and pairs of

fermions which will become massive at the SU(5) scale. One such suitable set consists of

$$6(\mathbf{495})+4(\overline{\mathbf{792}})+4(\overline{\mathbf{220}})+(\overline{\mathbf{66}})+4(\overline{\mathbf{12}}) \to 3(\mathbf{10}+\overline{\mathbf{5}}+\mathbf{1})+238(\mathbf{5}+\overline{\mathbf{5}})+211(\mathbf{10}+\overline{\mathbf{10}})+484(\mathbf{1}) \tag{1}$$

where the decomposition to anomaly-free SU(5) states has been indicated. The latter follows from the SU(12) $\to$ SU(5) branching rules:

$$\begin{aligned}
\mathbf{495} &\to 35(\mathbf{5})+21(\mathbf{10})+7(\overline{\mathbf{10}})+\overline{\mathbf{5}}+35(\mathbf{1}), && (2)\\
\overline{\mathbf{792}} &\to 7(\mathbf{5})+21(\mathbf{10})+35(\overline{\mathbf{10}})+35(\overline{\mathbf{5}})+22(\mathbf{1}), && (3)\\
\overline{\mathbf{220}} &\to \mathbf{10}+7(\overline{\mathbf{10}})+21(\overline{\mathbf{5}})+35(\mathbf{1}), && (4)\\
\overline{\mathbf{66}} &\to \overline{\mathbf{10}}+7(\overline{\mathbf{5}})+21(\mathbf{1}), && (5)\\
\overline{\mathbf{12}} &\to \overline{\mathbf{5}}+7(\mathbf{1}) && (6)
\end{aligned}$$

A search through the possible assignments of the three light chiral families to the SU(12) irreps appearing in the anomaly-free set of Eq. (1) reveals the following selection for a satisfactory low scale phenomenology:

$$\begin{array}{rrcl}
\text{1st Family}: & (\mathbf{10})\mathbf{495_1} & \supset & u_L,\ u^c_L,\ d_L,\ e^c_L\\
 & (\overline{\mathbf{5}})\overline{\mathbf{66_1}} & \supset & d^c_L,\ e_L,\ \nu_{1L}\\
 & (\mathbf{1})\overline{\mathbf{792_1}} & \supset & N^c_{1L}\\
\text{2nd Family}: & (\mathbf{10})\overline{\mathbf{792_2}} & \supset & c_L,\ c^c_L,\ s_L,\ \mu^c_L\\
 & (\overline{\mathbf{5}})\overline{\mathbf{792_2}} & \supset & s^c_L,\ \mu_L,\ \nu_{2L}\\
 & (\mathbf{1})\overline{\mathbf{220_2}} & \supset & N^c_{2L}\\
\text{3rd Family}: & (\mathbf{10})\overline{\mathbf{220_3}} & \supset & t_L,\ t^c_L,\ b_L,\ \tau^c_L\\
 & (\overline{\mathbf{5}})\overline{\mathbf{792_3}} & \supset & b^c_L,\ \tau_L,\ \nu_{3L}\\
 & (\mathbf{1})\overline{\mathbf{12_3}} & \supset & N^c_{3L}
\end{array} \tag{7}$$

Here the subscripts on the SU(12) irreps refer to the family in question, while the numbers in parentheses are just the SU(5) irreps chosen. Note that each SU(5) family multiplet can be uniquely assigned to a different SU(12) multiplet in the anomaly-free set according to (1). On the other hand, the remaining SU(5) multiplets are unassigned but form conjugate pairs which become massive and decouple at the SU(5) scale and are of no further interest.

EFFECTIVE THEORY APPROACH AND LEADING ORDER TREE DIAGRAMS

We start with the SU(12) model sketched above and take it to be supersymmetric. With a $\mathbf{143_H}$ adjoint Higgs field present, the breaking of SU(12) to SU(5) can occur via SU(12) $\to$ SU(5) $\times$ SU(7) $\times$ U(1), and in steps down to SU(5) via a set of antisymmetric chiral superfield irreps appropriately chosen to preserve supersymmetry [15, 16]. Unbroken supersymmetry at the SU(5) GUT scale allows us to deal only with tree diagrams in order to generate higher dimensional operators, for loop corrections are much suppressed.

For this purpose, we introduce massive $\mathbf{220}\times\overline{\mathbf{220}}$ and $\mathbf{792}\times\overline{\mathbf{792}}$ fermion pairs at the SU(12) scale. In addition, we introduce $(\mathbf{1})\mathbf{66_H}$, $(\mathbf{1})\overline{\mathbf{66_H}}$, and $(\mathbf{1})\mathbf{220_H}$, $(\mathbf{1})\overline{\mathbf{220_H}}$ conjugate Higgs pairs which acquire SU(5) singlet VEVs at the SUSY SU(5) GUT scale. Finally, doublets in $(\mathbf{5})\mathbf{924_H}$ and $(\overline{\mathbf{5}})\mathbf{924_H}$ Higgs fields effect the electroweak symmetry breaking at the electroweak scale. The list comprises then the following:

$$\begin{array}{llc}
\multicolumn{2}{c}{\text{Higgs Bosons}} & \text{Massive Fermions}\\
(\mathbf{5})\mathbf{924}_{\rm H} & (\overline{\mathbf{5}})\mathbf{924}_{\rm H} & \mathbf{220}\times\overline{\mathbf{220}},\\
(\mathbf{1})\mathbf{66}_{\rm H} & (\mathbf{1})\overline{\mathbf{66}}_{\rm H} & \mathbf{792}\times\overline{\mathbf{792}}\\
(\mathbf{1})\mathbf{220}_{\rm H} & (\mathbf{1})\overline{\mathbf{220}}_{\rm H} & \\
(\mathbf{24})\mathbf{143}_{\rm H} & &
\end{array} \tag{8}$$

For each element of the quark and lepton mass matrices, tree diagrams can then be constructed from three-point vertices which respect the SU(12) and SU(5) multiplication rules.

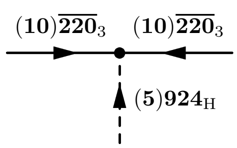
For illustration we present the lowest order tree diagram contributions to the 33 elements for the up and down quark mass matrices, taking into account the family assignments in (7). These are listed as **U33** and **D33**, respectively in (9).

Table 1. Leading order up- and down-type quark diagrams for each mass matrix element.

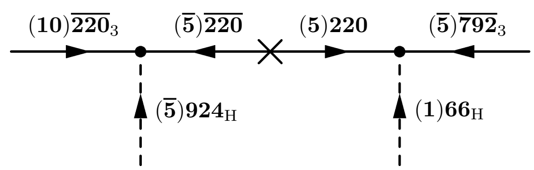
Up-Type Quark Mass-Term Diagrams		
Dim 4:	**U33:**	$(10)\overline{220_3}.(5)924_{\rm H}.(10)\overline{220_3}$
Dim 5:	**U23:**	$(10)\overline{792_2}.(1)66_{\rm H}.(\overline{10})220\times(10)\overline{220}.(5)924_{\rm H}.(10)\overline{220_3}$
	U32:	$(10)\overline{220_3}.(5)924_{\rm H}.(10)\overline{220}\times(\overline{10})220.(1)66_{\rm H}.(10)\overline{792_2}$
Dim 6:	**U13:**	$(10)495_1.(1)220_{\rm H}.(\overline{10})792\times(10)\overline{792}.(1)66_{\rm H}.(\overline{10})220\times(10)\overline{220}.(5)924_{\rm H}.(10)\overline{220_3}$
	U31:	$(10)\overline{220_3}.(5)924_{\rm H}.(10)\overline{220}\times(\overline{10})220.(1)66_{\rm H}.(10)\overline{792}\times(\overline{10})792.(1)220_{\rm H}.(10)495_1$
	U22:	$(10)\overline{792_2}.(1)66_{\rm H}.(\overline{10})220\times(10)\overline{220}.(5)924_{\rm H}.(10)\overline{220}\times(\overline{10})220.(1)66_{\rm H}.(10)\overline{792_2}$
Dim 7:	**U12:**	$(10)495_1.(1)220_{\rm H}.(\overline{10})792\times(10)\overline{792}.(1)66_{\rm H}.(\overline{10})220\times(10)\overline{220}.(5)924_{\rm H}.(10)\overline{220}\times(\overline{10})220$ $.(1)66_{\rm H}.(10)\overline{792_2}$
	U21:	$(10)\overline{792_2}.(1)66_{\rm H}.(\overline{10})220\times(10)\overline{220}.(5)924_{\rm H}.(10)\overline{220}\times(\overline{10})220.(1)66_{\rm H}.(10)\overline{792}\times(\overline{10})792$ $.(1)220_{\rm H}.(10)495_1$
Dim 8:	**U11:**	$(10)495_1.(1)220_{\rm H}.(\overline{10})792\times(10)\overline{792}.(1)66_{\rm H}.(\overline{10})220\times(10)\overline{220}.(5)924_{\rm H}.(10)\overline{220}\times(\overline{10})220$ $.(1)66_{\rm H}.(10)\overline{792}\times(\overline{10})792.(1)220_{\rm H}.(10)495_1$
Down-Type Quark Mass-Term Diagrams		
Dim 5:	**D32:**	$(10)\overline{220_3}.(\overline{5})924_{\rm H}.(\overline{5})\overline{220}\times(5)220.(1)66_{\rm H}.(\overline{5})\overline{792_2}$
	D33:	$(10)\overline{220_3}.(\overline{5})924_{\rm H}.(\overline{5})\overline{220}\times(5)220.(1)66_{\rm H}.(\overline{5})\overline{792_3}$
Dim 6:	**D31:**	$(10)\overline{220_3}.(\overline{5})924_{\rm H}.(\overline{5})\overline{220}\times(5)220.(1)66_{\rm H}.(\overline{5})\overline{792}\times(5)792.(1)\overline{220}_{\rm H}.(\overline{5})\overline{66_1}$
	D22:	$(10)\overline{792_2}.(1)66_{\rm H}.(\overline{10})220\times(10)\overline{220}.(\overline{5})924_{\rm H}.(\overline{5})\overline{220}\times(5)220.(1)66_{\rm H}.(\overline{5})\overline{792_2}$
	D23:	$(10)\overline{792_2}.(1)66_{\rm H}.(\overline{10})220\times(10)\overline{220}.(\overline{5})924_{\rm H}.(\overline{5})\overline{220}\times(5)220.(1)66_{\rm H}.(\overline{5})\overline{792_3}$
Dim 7:	**D12:**	$(10)495_1.(1)220_{\rm H}.(\overline{10})792\times(10)\overline{792}.(1)66_{\rm H}.(\overline{10})220\times(10)\overline{220}.(\overline{5})924_{\rm H}.(\overline{5})\overline{220}\times(5)220.(1)66_{\rm H}.(\overline{5})\overline{792_2}$
	D21:	$(10)\overline{792_2}.(1)66_{\rm H}.(\overline{10})220\times(10)\overline{220}.(\overline{5})924_{\rm H}.(\overline{5})\overline{220}\times(5)220.(1)66_{\rm H}.(\overline{5})\overline{792}\times(5)792.(1)\overline{220}_{\rm H}.(\overline{5})\overline{66_1}$
	D13:	$(10)495_1.(1)220_{\rm H}.(\overline{10})792\times(10)\overline{792}.(1)66_{\rm H}.(\overline{10})220\times(10)\overline{220}.(\overline{5})924_{\rm H}.(\overline{5})\overline{220}\times(5)220.(1)66_{\rm H}.(\overline{5})\overline{792_3}$
Dim 8:	**D11:**	$(10)495_1.(1)220_{\rm H}.(\overline{10})792\times(10)\overline{792}.(1)66_{\rm H}.(\overline{10})220\times(10)\overline{220}.(\overline{5})924_{\rm H}.(\overline{5})\overline{220}\times(5)220$ $.(1)66_{\rm H}.(\overline{5})\overline{792}\times(5)792.(1)\overline{220}_{\rm H}.(\overline{5})\overline{66_1}$

The convention is adopted that the left-handed fields appear on the left and the left-handed conjugate fields appear on the right.

U33:



D33:



(9)

For convenience we introduce the following short-hand notation to describe each of these diagrams:

$$\mathbf{U33}:\quad (\mathbf{10})\overline{\mathbf{220_3}}.(\mathbf{5})\mathbf{924_H}.(\mathbf{10})\overline{\mathbf{220_3}},\qquad \mathbf{D33}:\quad (\mathbf{10})\overline{\mathbf{220_3}}.(\overline{\mathbf{5}})\mathbf{924_H}.(\overline{\mathbf{5}})\overline{\mathbf{220}}\times(\mathbf{5})\mathbf{220}.(\mathbf{1})\mathbf{66_H}.(\overline{\mathbf{5}})\overline{\mathbf{792_3}}, \tag{10}$$

The leading order term for **U33** is seen to have dim-4, while that for **D33** has dim-5, due to the $(\mathbf{1})\mathbf{66_H}$ SU(5) Higgs singlet insertion resulting in one extra external Higgs field.

The full sets of leading order up- and down-quark diagrams for each matrix element is presented in Table I, while those for the Dirac and Majorana neutrino diagrams are listed in Table II. It is rather remarkable that only one diagram for each matrix element appears at leading order for all four mass matrices.

MASS MATRICES AND MIXINGS

Given the leading-order diagrams for each matrix element in Tables 1 and 2, we can then construct the quark and lepton mass matrices as follows. To each diagram corresponds a coupling constant or prefactor, h^u_{ij}, h^d_{ij}, h^{dn}_{ij} or h^{mn}_{ij} for the ijth element of the appropriate mass matrix, which is assumed to be of order one at the SU(12) unification scale, as naturalness predicts. Every SU(5) Higgs singlet insertion in higher-order tree diagrams introduces one power of

Table 2. Leading order Dirac and Majorana neutrino diagrams for each mass matrix element.

Dirac-Neutrino Mass-Term Diagrams		
Dim 4:	**DN23:**	$(\bar{5})\overline{792}_2.(5)924_{\mathrm{H}}.(1)\overline{12}_3$
	DN33:	$(\bar{5})\overline{792}_3.(5)924_{\mathrm{H}}.(1)\overline{12}_3$
Dim 5:	**DN13:**	$(\bar{5})\overline{66}_1.(1)\overline{220}_{\mathrm{H}}.(5)792\times(\bar{5})\overline{792}.(5)924_{\mathrm{H}}.(1)\overline{12}_3$
	DN22:	$(\bar{5})\overline{792}_2.(1)66_{\mathrm{H}}.(5)220\times(\bar{5})\overline{220}.(5)924_{\mathrm{H}}.(1)\overline{220}_2$
	DN32:	$(\bar{5})\overline{792}_3.(1)66_{\mathrm{H}}.(5)220\times(\bar{5})\overline{220}\ .(5)924_{\mathrm{H}}.(1)\overline{220}_2$
Dim 6:	**DN12:**	$(\bar{5})\overline{66}_1.(1)\overline{220}_{\mathrm{H}}.(5)792\times(\bar{5})\overline{792}.(1)66_{\mathrm{H}}.(5)220\times(\bar{5})\overline{220}.(5)924_{\mathrm{H}}.(1)\overline{220}_2$
	DN21:	$(\bar{5})\overline{792}_2.(1)66_{\mathrm{H}}.(5)220\times(\bar{5})\overline{220}.(5)924_{\mathrm{H}}.(1)\overline{220}\times(1)220.(1)66_{\mathrm{H}}.(1)\overline{792}_1$
	DN31:	$(\bar{5})\overline{792}_3.(1)66_{\mathrm{H}}.(5)220\times(\bar{5})\overline{220}.(5)924_{\mathrm{H}}.(1)\overline{220}\times(1)220.(1)66_{\mathrm{H}}.(1)\overline{792}_1$
Dim 7:	**DN11:**	$(\bar{5})\overline{66}_1.(1)\overline{220}_{\mathrm{H}}.(5)792\times(\bar{5})\overline{792}.(1)66_{\mathrm{H}}.(5)220\times(\bar{5})\overline{220}.(5)924_{\mathrm{H}}.(1)\overline{220}\times(1)220.(1)66_{\mathrm{H}}.(1)\overline{792}_1$
Majorana-Neutrino Mass-Term Diagrams		
Dim 4:	**MN11:**	$(1)\overline{792}_1.(1)\overline{66}_{\mathrm{H}}.(1)\overline{792}_1$
	MN33:	$(1)\overline{12}_3.(1)66_{\mathrm{H}}.(1)\overline{12}_3$
Dim 5:	**MN12:**	$(1)\overline{792}_1.(1)\overline{66}_{\mathrm{H}}.(1)\overline{792}\times(1)792.(1)\overline{66}_{\mathrm{H}}.(1)\overline{220}_2$
	MN21:	$(1)\overline{220}_2.(1)\overline{66}_{\mathrm{H}}.(1)792\times(1)\overline{792}.(1)\overline{66}_{\mathrm{H}}.(1)\overline{792}_1$
Dim 6:	**MN13:**	$(1)\overline{792}_1.(1)\overline{66}_{\mathrm{H}}.(1)\overline{792}\times(1)792.(1)\overline{66}_{\mathrm{H}}.(1)\overline{220}\times(1)220.(1)\overline{66}_{\mathrm{H}}.(1)\overline{12}_3$
	MN31:	$(1)\overline{12}_3.(1)\overline{66}_{\mathrm{H}}.(1)220\times(1)\overline{220}.(1)\overline{66}_{\mathrm{H}}.(1)792\times(1)\overline{792}.(1)\overline{66}_{\mathrm{H}}.(1)\overline{792}_1$
	MN22:	$(1)\overline{220}_2.(1)\overline{66}_{\mathrm{H}}.(1)792\times(1)\overline{792}.(1)\overline{66}_{\mathrm{H}}.(1)\overline{792}\times(1)792.(1)\overline{66}_{\mathrm{H}}.(1)\overline{220}_2$
Dim 7:	**MN23:**	$(1)\overline{220}_2.(1)\overline{66}_{\mathrm{H}}.(1)792\times(1)\overline{792}.(1)\overline{66}_{\mathrm{H}}.(1)\overline{792}\times(1)792.(1)\overline{66}_{\mathrm{H}}.(1)\overline{220}\times(1)220.(1)\overline{66}_{\mathrm{H}}.(1)\overline{12}_3$
	MN32:	$(1)\overline{12}_3.(1)\overline{66}_{\mathrm{H}}.(1)220\times(1)\overline{220}.(1)\overline{66}_{\mathrm{H}}.(1)792\times(1)\overline{792}.(1)\overline{66}_{\mathrm{H}}.(1)\overline{792}\times(1)792.(1)\overline{66}_{\mathrm{H}}.(1)\overline{220}_2$

$\varepsilon \equiv M_{\mathrm{SU}(5)}/M_{\mathrm{SU}(12)} \sim 1/50$ through the appearance of the ratio of the singlet Higgs VEV to the mass of the conjugate fermion fields after the latter are integrated out. Finally as a result of the electroweak spontaneous symmetry breaking, the $\mathbf{924_H}$ acquires a weak scale VEV, v. Hence for the two quark diagrams illustrated, the matrix element contributions are

$$\mathbf{U33}: h^u_{33}\mathrm{v}\, t^T_L t^c_L, \qquad\qquad \mathbf{D33}: h^d_{33}\varepsilon\mathrm{v}\, b^T_L b^c_L. \tag{11}$$

Note that as a result of the chiral SU(5) irrep structure, the lowest order tree diagram contribution to the 33 element of the charged lepton mass matrix is just the reflection of the diagram for the down quark 33 mass matrix element about the center of the diagram. Thus its 33 matrix element contribution is just the transpose of **D33**. More generally, the prefactors are related by $h^\ell_{ij} = h^d_{ji}$. By the same reasoning, it is clear that the up quark mass matrix elements are symmetric under interchange of i and j.

From Table 1 we then see that the two quark and charged lepton mass matrices are given by

$$\begin{aligned}
M_U &= \begin{pmatrix} h^u_{11}\varepsilon^4 & h^u_{12}\varepsilon^3 & h^u_{13}\varepsilon^2 \\ h^u_{12}\varepsilon^3 & h^u_{22}\varepsilon^2 & h^u_{23}\varepsilon \\ h^u_{13}\varepsilon^2 & h^u_{23}\varepsilon & h^u_{33} \end{pmatrix}\mathrm{v}\,, \\
M_D &= \begin{pmatrix} h^d_{11}\varepsilon^4 & h^d_{12}\varepsilon^3 & h^d_{13}\varepsilon^3 \\ h^d_{21}\varepsilon^3 & h^d_{22}\varepsilon^2 & h^d_{23}\varepsilon^2 \\ h^d_{31}\varepsilon^2 & h^d_{32}\varepsilon & h^d_{33}\varepsilon \end{pmatrix}\mathrm{v}\,, \\
M_L &= \begin{pmatrix} h^\ell_{11}\varepsilon^4 & h^\ell_{12}\varepsilon^3 & h^\ell_{13}\varepsilon^2 \\ h^\ell_{21}\varepsilon^3 & h^\ell_{22}\varepsilon^2 & h^\ell_{23}\varepsilon \\ h^\ell_{31}\varepsilon^3 & h^\ell_{32}\varepsilon^2 & h^\ell_{33}\varepsilon \end{pmatrix}\mathrm{v} = M_D^T.
\end{aligned} \tag{12}$$

While the up-quark matrix is symmetric, the down-quark and charged-lepton mass matrices are doubly lopsided in that the terms with h^d_{23} and h^l_{32} are suppressed by one extra power of ε compared with the h^d_{32} and h^l_{23} terms, respectively. For M_D, for example, this implies that a larger right-handed rotation than left-handed rotation is needed to bring the down quark matrix into diagonal form, while the opposite is true for M_L.

With the heavy right-handed neutrinos assigned to SU(5) singlets in (7), the resulting Dirac and Majorana neutrino 33 mass matrix elements receive the following dim-4 contributions as seen from Table 2:

$$\mathbf{DN33}: h_{33}^{dn}\mathrm{v}\overline{\nu}_{3L}N_{3L}^{c}, \qquad \mathbf{MN33}: h_{33}^{mn}\Lambda_R N_{3L}^{c^T} N_{3L}^{c}. \tag{13}$$

Here Λ_R represents the right-handed mass scale, typically of $\mathscr{O}(10^{14})$ GeV, whereas the SU(5) SUSY GUT scale is 2×10^{16} GeV to obtain gauge coupling unification. Again, a factor of ε enters for every singlet Higgs insertion in higher order diagrams. The two neutrino mass matrices can then be read off from Table 2, and we find

$$\begin{aligned}
M_{DN} &= \begin{pmatrix} h_{11}^{dn}\varepsilon^3 & h_{12}^{dn}\varepsilon^2 & h_{13}^{dn}\varepsilon \\ h_{21}^{dn}\varepsilon^2 & h_{22}^{dn}\varepsilon & h_{23}^{dn} \\ h_{31}^{dn}\varepsilon^2 & h_{32}^{dn}\varepsilon & h_{33}^{dn} \end{pmatrix}\mathrm{v}, \\
M_{MN} &= \begin{pmatrix} h_{11}^{mn} & h_{12}^{mn}\varepsilon & h_{13}^{mn}\varepsilon^2 \\ h_{12}^{mn}\varepsilon & h_{22}^{mn}\varepsilon^2 & h_{23}^{mn}\varepsilon^3 \\ h_{13}^{mn}\varepsilon^2 & h_{23}^{mn}\varepsilon^3 & h_{33}^{mn} \end{pmatrix}\Lambda_R.
\end{aligned} \tag{14}$$

where M_{DN} is also double lopsided, while M_{MN} is complex symmetric as usual. The symmetric light-neutrino mass matrix is obtained via the Type I seesaw mechanism:

$$M_\nu = -M_{\mathrm{DN}} M_{\mathrm{MN}}^{-1} M_{\mathrm{DN}}^{T}. \tag{15}$$

Keeping only the leading-order terms in ε for each matrix element, we find

$$M_\nu \approx \frac{\mathrm{v}^2}{\Lambda_R} \times \begin{pmatrix}
\varepsilon^2\left(\frac{h_{12}^{dn^2}h_{11}^{mn}}{h_{12}^{mn^2}-h_{11}^{mn}h_{22}^{mn}} - \frac{h_{13}^{dn^2}}{h_{33}^{mn}}\right) & \varepsilon\left(\frac{h_{12}^{dn}h_{22}^{dn}h_{11}^{mn}}{h_{12}^{mn^2}-h_{11}^{mn}h_{22}^{mn}} - \frac{h_{3}^{dn}h_{23}^{dn}}{h_{33}^{mn}}\right) & \varepsilon\left(\frac{h_{12}^{dn}h_{32}^{dn}h_{11}^{mn}}{h_{12}^{mn^2}-h_{11}^{mn}h_{22}^{mn}} - \frac{h_{13}^{dn}h_{33}^{dn}}{h_{33}^{mn}}\right) \\
\varepsilon\left(\frac{h_{12}^{dn}h_{22}^{dn}h_{11}^{mn}}{h_{12}^{mn^2}-h_{11}^{mn}h_{22}^{mn}} - \frac{h_{13}^{dn}h_{23}^{dn}}{h_{33}^{mn}}\right) & \frac{h_{22}^{dn^2}h_{11}^{mn}}{h_{12}^{mn^2}-h_{11}^{mn}h_{22}^{mn}} - \frac{h_{23}^{dn^2}}{h_{33}^{mn}} & \frac{h_{22}^{dn}h_{32}^{dn}h_{11}^{mn}}{h_{12}^{mn^2}-h_{11}^{mn}h_{22}^{mn}} - \frac{h_{23}^{dn}h_{33}^{dn}}{h_{33}^{mn}} \\
\varepsilon\left(\frac{h_{12}^{dn}h_{32}^{dn}h_{11}^{mn}}{h_{12}^{mn^2}-h_{11}^{mn}h_{22}^{mn}} - \frac{h_{13}^{dn}h_{33}^{dn}}{h_{33}^{mn}}\right) & \frac{h_{22}^{dn}h_{32}^{dn}h_{11}^{mn}}{h_{12}^{mn^2}-h_{11}^{mn}h_{22}^{mn}} - \frac{h_{23}^{dn}h_{33}^{dn}}{h_{33}^{mn}} & \frac{h_{32}^{dn^2}h_{11}^{mn}}{h_{12}^{mn^2}-h_{11}^{mn}h_{22}^{mn}} - \frac{h_{33}^{dn^2}}{h_{33}^{mn}}
\end{pmatrix} \tag{16}$$

which does not involve the prefactors h_{11}^{dn}, h_{21}^{dn}, h_{31}^{dn}, h_{13}^{mn} and h_{23}^{mn}.

The light-neutrino mass matrix exhibits a much milder hierarchy compared to the up-type and down-type mass matrices, as can be seen from the pattern of powers of ε. A mild or flat hierarchy of M_ν is conducive to obtaining large mixing angles and similar light neutrino masses. Furthermore, one observes that the light neutrino mass matrix obtained via the seesaw mechanism involves the doubly lopsided Dirac neutrino mass matrix twice. The lopsided feature of M_{DN} is such as to require a large left-handed rotation to bring M_ν into diagonal form.

NUMERICAL RESULTS

From the above up and down quark, charged lepton and light neutrino mass matrices, one can diagonalize the corresponding Hermitian matrices, $MM^\dagger$, in the usual manner to obtain the mass eigenvalues and the unitary transformations, U, effecting the diagonalizations. The Cabbibo-Kobayashi-Maskawa (CKM) quark mixing matrix and the corresponding Pontecorvo-Maki-Nakagawa-Sakata (PMNS) lepton mixing matrix then follow as

$$V_{CKM} = U_U^\dagger U_D \qquad V_{PMNS} = U_L^\dagger U_\nu \tag{17}$$

To obtain numerical results for the model predictions, we evaluate the mass matrices at the top quark mass scale and use just real prefactors to avoid too many fit parameters for good fit convergence. There are 6 prefactors each for the symmetric up quark and Majorana matrices, 9 each for the lopsided down quark and Dirac neutrino matrices, but 5 of them do not appear in the light neutrino mass matrix, making a total of 25 parameters. In addition, we have one for the right-handed neutrino scale, Λ_R plus a value for ε which we fix at $\varepsilon = 1/6.5^2 = 0.0237$ again for good fit convergence, for a grand total of 26 adjustable fit parameters. To avoid correlated data parameters, we make use of the 9 quark and charged lepton masses plus the 3 neutrino Δm^2's and the 18 CKM and PMNS mixing parameters taken to be real, for a total of 30 data points. We refer the reader to our published paper [13] for full details of the fitting procedure, where

we have included the latest best value for the reactor neutrino mixing angle, θ_{13}. There can be found a table giving the phenomenological mass and mixing data entering the fit, as well as the theoretical mass and mixing results obtained from the fitting procedure.

The best fit was obtained with a normal neutrino mass hierarchy with $\Lambda_R = 7.4 \times 10^{14}$ GeV and the following mass matrices:

$$
\begin{aligned}
M_U &= \begin{pmatrix} -1.1\epsilon^4 & 7.1\epsilon^3 & 5.6\epsilon^2 \\ 7.1\epsilon^3 & -6.2\epsilon^2 & -0.10\epsilon \\ 5.6\epsilon^2 & -0.10\epsilon & -0.95 \end{pmatrix} \mathrm{v}, & M_D &= \begin{pmatrix} -6.3\epsilon^4 & 8.0\epsilon^3 & -1.9\epsilon^3 \\ -4.5\epsilon^3 & 0.38\epsilon^2 & -1.3\epsilon^2 \\ 0.88\epsilon^2 & -0.23\epsilon & -0.51\epsilon \end{pmatrix} \mathrm{v} = M_L^T, \\
M_{DN} &= \begin{pmatrix} h^{dn}_{11}\epsilon^3 & 0.21\epsilon^2 & -2.7\epsilon \\ h^{dn}_{21}\epsilon^2 & -0.28\epsilon & -0.15 \\ h^{dn}_{31}\epsilon^2 & 2.1\epsilon & 0.086 \end{pmatrix} \mathrm{v}, & M_{MN} &= \begin{pmatrix} -0.72 & -1.5\epsilon & h^{mn}_{13}\epsilon^2 \\ -1.5\epsilon & 0.95\epsilon^2 & h^{mn}_{23}\epsilon^3 \\ h^{mn}_{13}\epsilon^2 & h^{mn}_{23}\epsilon^3 & 0.093 \end{pmatrix} \Lambda_R, \\
M_\nu &= \begin{pmatrix} -81\epsilon^2 & -4.3\epsilon & 2.4\epsilon \\ -4.3\epsilon & -0.25 & 0.28 \\ 2.4\epsilon & 0.28 & -1.1 \end{pmatrix} \mathrm{v}^2/\Lambda_R. & &
\end{aligned}
\tag{18}
$$

Note that all prefactors except three in the above matrices are within a factor of 0.1 to 10 of unity for this best fit. The five independent prefactors, h^{dn}_{11}, h^{dn}_{21}, h^{dn}_{31}, h^{mn}_{13} and h^{mn}_{23}, do not influence the fit and remain undetermined as noted earlier. For this best fit we find the neutrino mass values

$$
m_1 = 0, \quad m_2 = 8.65, \quad m_3 = 49.7 \text{ meV}; \qquad M_1 = 1.67 \times 10^{12}, \quad M_2 = 6.85 \times 10^{13}, \quad M_3 = 5.30 \times 10^{14} \text{ GeV}. \tag{19}
$$

In addition, the best fit favors $\delta_{CP} = \pi$ for the leptonic CP Dirac phase. The value of ϵ used then implies that the SU(12) GUT scale is about $M_{\mathrm{SU(5)}}/\epsilon = 8.4 \times 10^{17}$ GeV, just below the reduced Planck scale, where we have used 2×10^{16} GeV for the SU(5) unification scale.

All remaining mass and mixing parameters are fit quite well by the model; however, since M_L is just the transpose of M_D in leading order in ϵ, the Georgi-Jarlskog relations [17] are not satisfied for the down quarks and charged leptons. We have checked that the addition of an adjoint $\mathbf{143}_{\mathrm{H}}$ Higgs field whose VEV points in the $B-L$ direction contributes to M_D and M_L at one higher order of ϵ, so that the $M_L = M_D^T$ relation is broken, and more accurate values can be obtained for the down quark and charged lepton mass eigenvalues.

SUMMARY

A unified SU(12) SUSY GUT model was obtained by a brute force computer scan over many SU(12) anomaly-free sets of irreps containing 3 SU(5) chiral families under the assumption that the symmetry is broken in stages from SU(12) $\rightarrow$ SU(5) $\rightarrow$ SM. In doing so, looping over all SU(12) fermion and Higgs assignments was performed with good fits to the input data required. For this purpose an effective theory approach was used to determine the leading order tree-level diagrams for the dim-(4 + n) matrix elements in powers of ϵ^n, where ϵ is the ratio of the SU(5) to the SU(12) scale. The best fit was obtained by requiring all prefactors to be $\mathscr{O}(1)$, but the large number of them implies just a few predictions. With no discrete flavor symmetry adopted, problems with breaking by gravity, domain walls and explanation of its origin can be avoided [18, 19, 20]. On the contrary, with such a large SU(N) gauge group, a host of heavy fermions is predicted which are integrated out at the SU(5) scale.

The SU(12) model considered is just one of many possibilities (including other assignments and larger SU(N) groups), but its features were among the most attractive found: Each SU(5) family supermultiplet can be assigned to a different SU(12) multiplet in the anomaly-free set. In the model considered, only one diagram appears for each matrix element for all 5 mass matrices, but some additional contribution is needed to obtain the Georgi-Jarlskog relations.

Among some distracting features we point out the prefactors are determined at the top quark scale. They should be run to the SU(5) unification scale to test their naturalness. The fit considers only real prefactors, so CP violation is not accommodated, but the fit preferred $\delta_{CP} = \pi$ over $\delta_{CP} = 0$ for the leptonic CP phase. The complete breaking of SU(12) $\rightarrow$ SU(5) while preserving supersymmetry needs to be worked out in more detail and is under further study.

ACKNOWLEDGMENTS

One of us (CHA) thanks Kaladi Babu, Rabi Mohapatra, and Barbara Szczerbinska for the kind invitation to attend and present a talk on this work at the CETUP* Workshop on Neutrino Physics and Unification in Lead, SD in July 23 - 29, 2012. He especially appreciated some constructive suggestions by participants at the Workshop. He thanks the Fermilab Theoretical Physics Department for its kind hospitality, where part of this work was carried out. The work of RPF was supported by a fellowship within the Postdoc-Programme of the German Academic Exchange Service (DAAD). The work of RPF and TWK was supported by US DOE grant E-FG05-85ER40226. Fermilab is operated by Fermi Research Alliance, LLC under Contract No. De-AC02-07CH11359 with the U.S. Department of Energy.

REFERENCES

1. G. Altarelli and F. Feruglio, Rev. Mod. Phys. **82**, 2701 (2010), arXiv:1002.0211 [hep-ph].
2. H. Ishimori, T. Kobayashi, H. Ohki, Y. Shimizu, H. Okada, et al., Prog.Theor.Phys.Suppl. **183**, 1 (2010), arXiv:1003.3552 [hep-th].
3. M. Gell-Mann, P. Ramond, and R. Slansky, Conf. Proc. C790927, 315 (1979), in Supergravity, P. van Nieuwenhuizen and D.Z. Freedman (eds.), North Holland Publ. Co., 1979.
4. Y. Fujimoto, Phys.Rev. D**26**, 3183 (1982).
5. H. Georgi, Nucl.Phys. B**156**, 126 (1979).
6. J. E. Kim, Phys.Rev. D**24**, 3007 (1981).
7. P. Frampton, Phys.Lett. **B88**, 299 (1979).
8. P. Frampton and S. Nandi, Phys.Rev.Lett. **43**, 1460 (1979).
9. S. Barr, Phys. Rev. D**78**, 055008 (2008), arXiv:0805.4808 [hep-ph].
10. S. Barr, Phys. Rev. D**78**, 075001 (2008), arXiv:0804.1356 [hep-ph].
11. P.H. Frampton and T.W. Kephart, Phys. Lett. **B681**, 343 (2009), arXiv:09004.3084 [hep-ph].
12. J.B. Dent, R. Feger, T.W. Kephart, and S. Nandi, Phys. Lett **B697**, 367 (2011), arXiv:0908.3915 [hep-ph].
13. C.H. Albright, R. Feger, and T.W. Kephart, Phys. Rev. D**86**, 015012 (2012), arXiv:1204.5471 [hep-ph].
14. R. Feger and T.W. Kephart, arXiv:1206.6379 [hep-ph].
15. P.H. Frampton and T.W. Kephart, Phys. Rev. Lett. **48**, 1237 (1982).
16. P. Frampton and T. Kephart, Nucl. Phys. **B211**, 239 (1983).
17. H. Georgi and C. Jarlskog, Phys. Lett. **B 86**, 297 (1979).
18. R. Holman, S.D. Hsu, T.W. Kephart, E.W. Kolb, R. Watkins, and L.M. Widrow, Phys. Lett. **B282**, 132 (1992), arXiv:hep-ph/9203206 [hep-ph].
19. M. Kamionkowski and J. March-Russell, Phys. Lett. **B282**, 137 (1992), arXiv:hep-th/9202003 [hep-th].
20. S.M. Barr and D. Seckel, Phys. Rev. D**46**, 539 (1992).